LPS WRS Horizontal Display Shift Algorithm

Problem Statement:

Given the locations of 2 nominal scene centers and the actual scene center find the horizontal display shift (HDS) in kilometers on the geoid. The HDS is defined as the perpendicular distance of the actual scene center from the nominal scene center track.

Solution

The inputs required for this function are the latitudes and longitudes of the actual scene center and 2 nominal scene centers, and the spacecraft velocity vector at the time of the actual scene center. Before finding the HDS it is necessary to find the perpendicular angular distance on the unit sphere from the actual scene center to the nominal track. This is given by

$$\boxed{d = \cos^{-1}[\hat{N} \cdot \hat{R}] - \frac{\pi}{2}}$$

Where :

$\hat{R} = (\alpha_a, \delta_a)$ is the actual scene center

$\hat{C}_1 = (\alpha_1, \delta_1)$ is the nominal scene center nearest the actual center ($\mathrm{Path}_1, \mathrm{Row}_1$)

$\hat{C}_2 = (\alpha_2, \delta_2)$ is the previous nominal scene center ($\mathrm{Path}_1, \mathrm{Row}_1 - 1$)

$\hat{N} = (\alpha_{\mathrm{normal}}, \delta_{\mathrm{normal}})$ is the normal vector formed from $\hat{C}_1$ and $\hat{C}_2$

(α, δ) is longitude and geocentric latitude

The vectors to each of the three scene centers are formed from the longitude and latitude as follows (using the appropriate subscripts for each):

$$\begin{bmatrix} \cos(\delta)\cos(\alpha) \\ \cos(\delta)\sin(\alpha) \\ \sin(\delta) \end{bmatrix}$$

The unit normal is obtained from

$$\hat{N} = \frac{\hat{C}_2 \times \hat{C}_1}{\left\| \hat{C}_2 \times \hat{C}_1 \right\|}$$

Note that d can be either positive or negative. The sign of d is a function of the relative east/west position of the actual scene center to that of the nominal track. This relationship can be expressed as a Boolean function, East_True, using the sign of the z-component of velocity (V_z) and the sign of d.

$$\boxed{East\_True = (1 + sign(V_z) \cdot sign(d)) / 2}$$

East_True = 1 means that the actual scene center is east of the nominal track whereas East_True = 0 implies that it is west of the nominal track.

Using the radius of the Earth at the latitude of the actual scene center we can calculate the value of the HDS in km on the Earth's surface:

$$\boxed{r(\delta_a) = \frac{a \cdot b}{\sqrt{a^2 \sin^2(\delta_a) + b^2 \cos^2(\delta_a)}}}$$

$$\boxed{HDS = r(\delta_a) \cdot d}$$

Where :
a is the equatorial radius of the Earth
b is the polar radius of the Earth